Conceptualizing the Emergence of Entropy

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ABSTRACT

In the following we discuss two major approaches to the conceptualization of entropy in terms of the concept of emergence. The classical viewpoint of visualizing entropy as a macroscopic quantity measuring the degree of deficient knowledge is compared with the idea of visualizing entropy as a measure of some intrinsic (chaotic) indeterminism. The respective line taken leads forward to significant epistemological consequences which lie at the foundation of the relationship between macrophysics and microphysics.

Keywords: information entropy, locality, quantum gravity

1. INTRODUCTION

As to the concepts of *emergence* and *complexity* which have become prominent in the sciences lately, the common opinion is dominating to ascribe them to the human mode of cognitively modeling the physical world rather than to this world itself. In classical presentations of thermodynamics this aspect is usually based on a similar viewpoint as put forward by Boltzmann for whom quantities such as temperature, entropy etc. are chiefly macroscopic quantities only and cannot be actually found on the microscopic level proper. The temperature of a solid or fluid body or a gas e.g. shows up as observable quantity on the macroscopic level and can thus be easily measured, but on the molecular level it turns out to be equated with the average kinetic energy of the molecules constituting the body or gas of which the temperature is actually being measured: $\frac{1}{2}\, m\, c^2 = 3/2\, (Rm/M)\, T = 3/2\, k\, T$, where c is the average velocity of the molecules, R is the gas constant, and M the molar mass (Bergmann et al. 1970; Sommerfeld 1977; Crutchfield 1994; Edmonds 1999). It is this formal equality however that usually veils the fact that the temperature on the right-hand side belongs to a level which is macroscopic with respect to the left-hand side. In other words, "temperature" is simply a conventional *name* for the measuring of a phenomenon which can be observed only in terms of a mean value. In this sense, humans are "short-sighted" and utilize the name for practical reasons. Hence, the concept of temperature is *emergent* with respect to the particle statistics actually producing a phenomenon (heat) that can be observed and measured by a quantity of that name. In turn, masses and velocities on their own molecular level are emergent concepts with respect to the next lower level of fundamental particles, and so forth, all the way down to quark and leptons, say.

The same attitude should be valid in principle, if visualizing the other quantites of classical thermodynamics which show up e.g. in the free energy expression $F = U - TS$, or in the free enthalpy expression $G = H - TS$, respectively. Among them is entropy. Hence, under this systematic perspective, the concept of emergence is deeply connected to the concept of information. Essentially, we can say that the state space representation replacing the configurational representation of classical thermodynamics leads to conceptualizing entropy within statistical physics as a state property and a measure of deficient knowledge, respectively, rather than as observable (in the sense of operators on Hilbert spaces). This is consistent within the Boltzmannian viewpoint, but not so in the sense of Prigogine's approach: Prigogine tries to resolve the common debate on the contrast of reversibility and indeterminism of microscopic laws (including quantum theory) on the one hand and irreversibility and determinism of macroscopic laws on the other by advocating (contrary to Boltzmann) that irreversibility and thus entropy are not limited to macroscopic physics. In fact, for him, processes like the decay of an excited state contribute to the entropy balance as well. Moreover, contrary to Pauli, Prigogine also purports that the symmetry between incoming and outgoing waves as used in S-matrix theory is broken, because the incoming waves have lower entropy. And finally, he also tries to actually unify (i.e. sublate) the duality between reversible and irreversible processes introduced originally by von Neumann. The consequence is then that *probability* does not emerge from supplementary approximations made because of lack of knowledge, but instead as a dynamical consequence of resonance singularities in non-integrable systems. (This is somehow reminiscent of the way Penrose once introduced his twistor concept.) In Prigogine, irreversibility is thus being formulated within a theory of transformations that expresses in explicit terms what the classical formulation of the dynamics tends to hide (Karpov et al. 2003).

2. PRIGOGINE's APPROACH

To be more precise Prigogine locates the reasons for the classical approach in the limitation to integrable systems. For these systems there is a unitary transformation U which is distributive:

$$U(ab) = (Ua)\,(Ub).$$

But for *non-integrable* systems (which form the majority in nature), the unitary operator has to be replaced by a more general operator Λ called *star unitary* which is not distributive:

$$\Lambda(ab) \neq (\Lambda a)\,(\Lambda b).$$

From time evolution as a unitary group we pass then to Markovian dynamics by means of utilizing a Lyapunov variable $H := \Lambda^* \Lambda$ (the star indicating hermitian conjugacy). The operator Λ is displayed then as giving rise to a *non-local* transformation which means that in the new representation, points are being replaced by ensembles. Contrary to Planck (in the Hilbert space representation) there is a microscopic formulation of thermodynamics now which includes decay and excitation of quantum states. Entropy creation is here due to a resonance in the time evolution. The Heisenberg evolution is

$$H(t) = \exp(iht)\, H \exp(-iht) = \exp(-2\gamma t)\, H,\ \gamma > 0,$$

where h is the usual Hamiltonian. The expectation value of H decreases, hence entropy increases as the energy of the excited state is transferred to the field modes. For more details see Karpov et al. (2003).

In the language of density matrices, Prigogine replaces the common entropy functional $-k\,\mathrm{Tr}\,\rho \ln \rho$ by introducing a modified $\rho' = \Lambda^{-1}(L)\,\rho$ which is a superoperator on Hilbert space of the system in question defined as a function of the Liouville operator such that the von Neumann equation for ρ, namely $d\rho/dt = -i L \rho$, induces an equation of motion for ρ':

$$d\rho'/dt = -i\,\Lambda^{-1}(L)\, L\, \Lambda(L)\, \rho'$$

which thus defines a new entropy functional of the form $-k\,\mathrm{Tr}\,\rho' \ln \rho'$ (and which increases with time) (Beretta 2008).

The roots of this approach have been layed down as early as 1979 (Prigogine 1979, 1993, 1997) and it is astonishing to note that by doing so, Prigogine replaces unitarity by non-unitarity and actually leaves the representations of Hilbert space. The point is that he is able to discuss mixed systems in the sense that reversible and irreversible components of a generalized evolution operator can be unified such that the symmetry properties reproduce those of Boltzmann's equation. Not only is the underlying entropy operator of *microscopic* kind, but it can be also explicitly written as a product of some operator T with its hermitian adjoint such that T turns out to be equal to Λ^{-1} and can be interpreted as *time operator*. In fact, it can be shown that macroscopic time is nothing but the mean value of this new time operator.

This can be understood when thinking of Prigogine's version of the uncertainty relation which involves the Liouville operator:

$$[L, T] = iI.$$

Then define the mean values $\langle T \rangle$ and $\langle T^2 \rangle$ by Tr $(\rho^* \; T \; \rho)$ and Tr $(\rho^* \; T^2 \; \rho)$, respectively. (Note that again, exceptionally, we utilize the star here for hermitian conjugates.) We find by the above uncertainty relation that

$$d/dt \langle T \rangle = \mathrm{Tr}\,(\rho^* \rho) = \mathrm{const.}$$

Normalization such that the constant is 1 gives thus:

$$dt = d \langle T \rangle.$$

3. BLACK HOLE ENTROPY

In approaches to quantum gravity, notably to *loop quantum gravity*, the role of spin networks becomes relevant (Ashtekar et al. 2000). In particular, it is the edges of spin networks that pierce the horizon of black holes and by doing so excite curvature degrees of freedom on the surface. These excitations are microscopic states that account for the black hole entropy after all. We can thus find arguments in favour of visualizing a black hole as a device which is encoding quantum information (Zizzi 2000, 2005, 2006).

When discussing black holes, it is important to determine the appropriate density matrix on the physical part of Hilbert space which corresponds to the maximal entropy mixture of surface states for which the horizon area lies in this range.

The respective entropy is then

$$S_{bh} = - \mathrm{Tr}\,(\rho_{bh} \ln \rho_{bh})$$

and thus

$$S_{bh} = \ln N_{bh},$$

with N being the dimension of the black hole's Hilbert space which is the number of physical surface states. Those states have to be counted which form a basis of that Hilbert space. Then, according to the computation shown by Ashtekar et al (2000), we find

$$S_{bh} = \ln 2 / 4\pi\sqrt{3}\gamma l_p^2\, a_0 + o(a_0).$$

If γ is set to $\ln 2/\pi\sqrt{3}$ to give the Barbero-Immirzi parameter, then the Hawking-Bekenstein result is being reproduced. For the black hole temperature and entropy, respectively, we find in the standard approach

$$T = \hbar/2\pi\,\kappa, \quad S = 1/4l_p^2\, a.$$

Here, κ refers to the surface gravity, i.e. to that acceleration which is necessary to keep objects at the horizon, and a is the surface area of the black hole's horizon, while l_p refers to the Planck length. (For a general Kerr-Newman black hole e.g., we would have $\kappa = (r_+ - r_-)/2(r_+^2 + a^2)$, where the r mean the two horizons $r_\pm = m \pm (m^2 - q^2 - a^2)^{1/2}$ with m as mass, q as electric charge, and a as angular momentum. We find then the area as $A = 8\pi\, m\, (m + (m^2 - q^2 - a^2)^{1/2}$, and the entropy as $S = 2\pi k/\hbar\, [m\, (m + (m^2 - q^2 - a^2)^{1/2}$, respectively.) Wheeler's "It-from-Bit" philosophy purports then that the dominant contribution to entropy comes from states in which there is a very large number of punctures each labelled by j = ½ (half spin) and a = ± 1. This has been generalized with respect to quantum computation by Paola Zizzi (2000, 2005, 2006). Ashtekar et al. conclude from this that "[e]ntropy thus depends on the division of spacetime into exterior and interior regions, and is not an intrinsic attribute of spacetime. It is tied to the class of observers who live in the exterior region and for whom the isolated horizon is a physical boundary that separates the part they can access from that they cannot."

4. KAUFFMAN's VIEWPOINT

If we follow Stuart Kauffman's viewpoint of visualizing processes in nature as *co-operative games of universal autonomous agents*, then the underlying entropy concept leads to a generalization of the thermodynamic laws. Hence, following the idea of Kauffman's, we think of *agents* in the generalized sense as systems which achieve a new kind of closure in a given space of catalytic and work tasks propagating work out of non-equilibrium states and playing natural games according to the constraints of their environment (Kauffman 2000). In particular, (physical) space is visualized as being comprised of autocatalytic autonomous Planck scale agents co-evolving with each other serving at the same time as some sort of crystallization of seeds of classicity (in the physical sense). This co-evolution is taking place according to what Kauffman calls the *4th law of thermodynamics*: The maximum growth of the adjacent possible in the flow of a non-ergodic Universe maximizes the rate of decoherence and thus the emergence of classicity. There is also a hierarchy of such agents depending on the explicit complexity of those in question ("higher-order agents") such that *human agents* in particular (as components of social systems) represent a stage of higher complexity as compared to physical, chemical, or biological systems. But on the fundamental level of physics, Kauffman mentions the possibility to visualize *spin networks* as knots acting on knots to create knots in rich coupled cycles not unlike a metabolism. Hence, they (or their constituents) show up as a sort of "fundamental agents".

If we take up this viewpoint, then it appears to be straightforward to find instead the fundamental agents in the loops proper of *loop quantum gravity* (and the associated *quantum information theory*) in the first place, which again would be consistent with Zizzi's approach: This is so because it is the loops which combine in order to form spin networks. With a *loop* we mean here a closed curve α such that $T[\alpha] = -\text{Tr}\,[U_\alpha]$, where

$$U_\alpha\,(s_1, s_2) \sim P\,\exp\,\{\textstyle\int^{s2}_{s1} A_\alpha\,(\alpha(s))\,ds\}$$

is the parallel propagator of A_α along α defined by (the s_i being points of α)

$$d/ds\,U_\alpha\,(1, s) = da_i(s)/ds\,A_i(i(s))\,U_\alpha\,(1, s).$$

The SO(3)-field A is here essentially the difference of the SU(2)-spin connection and the extrinsic 3-curvature called *real Ashtekar connection*:

$$A_i^j\,(x) = \Gamma_i^j\,(x) - k_i^j\,(x).$$

The important result (cf. Rovelli) is that each spin network state can be decomposed into a finite linear combination of products of loop states.

Our conjecture is then that *loops are universal agents*. In other words, six of them, respectively, recombine to give one compartment of the hexagonal structure spanned by the spin networks. The conceptual reason for this is that the associated entropy satisfies the criterion for a thermodynamic cycle process such that

$$1/4l_p^2 \int da \leq 0,$$

where the integral is a closed path integral and a is the surface area generated with respect to one hexagonal fragment of the spin network. By the definition of loops above we clearly recognize that this procedure is not referring to some physically "vacuous" geometrical meaning, but that instead, this geometrical picture is physically loaded due to the parallel propagator with its gravitational or curvature connotation, respectively, and the explicit group action involved.

Obviously, this bears a strong resemblance to the Wilson loop representation (hence, we think here of a kind of *loop transport* according to Stuart Kauffman's idea of agents), and is also essentially a Feynman-type integral which gives the probability for a (physical) system to go from one state to another:

$$< x2, t2 \mid x1, t1 > = \int^{XN}_{X1} D(x(t)) \exp i/\hbar S,$$

where S is he action of the form

$$S := \int^{t2}_{t1} dt\, L(x, x'),$$

The probability is the above expression squared. This is equivalent to the Schroedinger picture of quantum physics on the one hand and a model for quantum computation on the other. As Freidel and Krasnov (1999) as well as Reisenberger and Rovelli (2000) have shown, spin networks and spin foams, respectively, can be visualized as Feynman integrals of that sort such that the formal Feynman perturbation series of the partition function

$$Z = \int D\phi \exp(-S[\phi])$$

is given by

$$Z = \Sigma_J N(J) \Sigma_e \Pi_{f\in J} \dim a_f \Pi_v A_v(e),$$

where J is a 2-complex, and the vertices, edges, and faces are labelled accordingly. It is N(J) the number of vertices of J divided by the number of symmetries of J.

There is a number of important cross-relationships which connect the notion of loops with the notion of knots: Louis Kauffman's bracket algebra (the boundary algebra of containers and extainers) turns out to be the precursor of the Temperley-Lieb algebra important in order to construct representations of the Artin braid group related to the Jones polynomial in the theory of knot invariants. As the elementary bracket algebra is to *biologic* what Boolean logic is to classical logic, this has important epistemological consequences (on whose biosemiotic aspects I have reported in Salzburg last year (Zimmermann, *tripleC* 2007)). On the other hand, the Jones polynomial can itself be visualized in terms of quantum computers, because a similar partition function of the form $Z_K = < \text{cup} \mid M \mid \text{cap} >$ with creation and annihilation operations, respectively,

$$\text{cup} := \mid a > : C \rightarrow V \otimes V,$$

$$\text{cap} := < b \mid : V \otimes V \rightarrow C,$$

M being the braiding, and $< K > := \Sigma_\sigma < K \mid \sigma > d^{||\sigma||}$ being related to the process of quantum computation (similar to, by the way, the spin network formalism itself.) As spin networks are nothing but graphs, the *agency* in question here is motion on graphs or percolation in networks such that phase transitions can be represented in terms of an appropriate cluster formation of connected components. This also points towards a close relationship to cellular automata utilized for the simulation of evolutionary processes (cf. Conway's game of life). Stuart Kauffman has associated this with the emergence of collectively autocatalytic sets of polymers, and in fact with the onset of forming classicity with regards to physics. It is straightforward (in epistemic terms) to generalize this (with a view to higher-order agents) to chemical, biological, and other systems. This is very much on the line of Zizzi (2000, 2005, 2006).

Utilizing the "skeleton-of-the-universe view" described in my DPG spring conference talk (Zimmermann, *Zeitspruenge* 2007), the idea would be to insert various steps of a hierarchy of complexity in the overall functor diagram from topological quantum field theory (cf. John Baez):

$$\begin{array}{ccc} \text{nCob} & \rightarrow & \text{Hilb} \\ \uparrow em & & \uparrow\downarrow id \\ \text{SpinF} & \rightarrow & \text{Hilb} \end{array} .$$

This diagram is commutative, if an adequate emergence (em) mapping is being defined. Here, SpinF is the category of spin foams, and nCob is the category of n-dimensional cobordisms. (For the time being, we can safely set n = 4.)

5. CONCLUSION

What is actually being confronted when comparing the above-mentioned approaches, is the idea (as in classical thermodynamics as well as loop quantum gravity indicating the most fundamental level of physics) that the hierarchically ordered "averaging out" of indeterministic and reversible phenomena on the small scale is producing well-determined and irreversible phenomena on the large scale of the physical world on the one hand – and the different idea (as in Prigogine's approach) that even on the lowest accessible level of the physical world, there is a concrete and active mixture of deterministic and indeterministic processes such that even intrinsically unstable fluctuations can actually create and determine large-scale phenomena on the other. The first is a chiefly *epistemological* argument which ties the results obtained to the method applied. In a sense, this can be visualized as an almost Spinozist viewpoint. The second is a more *ontological* argument detecting a creative potential of nature itself on its most fundamental level which ties the results obtained to some objective "form of being". This can be visualized as a Bergsonian viewpoint. The first entails the "metaphysical" idea that knowledge will be permanently increased and phenomena will become more and more observable. The second entails that on the microlevel, phenomena are non-observable in principle. Indeed, the mentioned philosophers, and those who derived their approaches from one or the other, respectively, did actually influence various scientists belonging to the one or other group. In particular, it is Prigogine who draws heavily on the ideas of Bergson who in turn was known for his famous debate with Einstein as to that point. However, in recent research we find various strings of argument which favour the first viewpoint or equally, the second, according to the long-term perspective taken: Whether something can be observed or not *in principle* is decided here according to whether one takes the strict opinion that true observation can only be based on sensory perception, or the generalized opinion that observation encompasses all what instruments can actually measure, and if so, whether observation is objective or not (Let us attribute the latter conviction to the classical viewpoint described above.)

Hence, the classical viewpoint makes clear that what can be observed (and known) depends on the evolutionary (and transitory) state of human knowledge, while Prigogine would argue instead that knowledge is limited *in principle*, because some details of nature are not yet settled by nature itself. (This is, by the way, an argument which places Prigogine in the vicinity of the philosopher Ernst Bloch.) The technical arguments involved are equally elegant which is particularly obvious when discussing the most fundamental level of physics, that of quantum gravity. On that level, the introduction of an explicit reference to human knowledge gains a global kind of cosmological relevance.

There is however, a major conceptual difficulty in the arguments of both viewpoints: In fact, in some sense, we would like to agree to both of them – but contrary somehow to their own arguments. Given the fact that human sensory perception is incomplete in the first place, we have no other choice than sharing the position taken by Prigogine. This is mainly so, because we would not like to agree to the assumption that instruments are merely means of "generalized sensory perception", but instead we would argue that they are constructed within the frame of reference given by existing theories in the first place. Which also implies that for us, observations are subjective in the anthropological sense. Hence, we share Kuhn's conception dealing with paradigms (Zimmermann 2002, 2005, 2006). This implies then "architectural" consequences as to the relationship between the theories developed and the phenomena observed (Sambasivam & Bodas 2006, Majima & Suzuki 2005). On the other hand, taking this for granted, it does not entail anything which would speak against the viewpoint of "averages". Hence, we could also agree with the classical position as far as it goes with respect to averages. Therefore, the situation is such:

	epistemological view	*ontological view*
	(Boltzmann, …, Ashtekar etc.)	(Prigogine etc.)
macrophysics	irreversible, deterministic average observable	mixture observable
microphysics	reversible, indeterministic details partially observable	mixture non-observable
metaphysical frame of reference	entails increasing observability	entails non-observability in principle

In other words, even if human beings might be able one day to possess complete knowledge with respect to their frame of reference which is determined by their biological capacity of sensory perceptions and including the development of instruments, their knowledge will only partly describe the world *as it really is*. In other words, we favour the viewpoint of Prigogine *while argueing ontologically*, but this does not necessarily imply Prigogine's conclusion on the relationship between what is observable and what is non-observable, because he refers to internal knowledge (within the interior of the world) rather than to external knowledge (of the world altogether). But we favour the classical viewpoint *while argueing epistemologically*. Indeed, our position is rather a compromise, we can call it *onto-epistemological* (Sandkuehler), and we indeed refute the classical viewpoint of an objective knowledge which can be eventually completed.

A scientific compromise might offer Kauffman's approach: If we accept his viewpoint for a moment, then all what he says is compatible with the idea of humans being complex agents organized in complex communities of agents playing natural games which in their case specialize to *social* games and are called meaningful. The cognitive activity of humans then, thinking, and modelling the world, is a complex activity defined according to their complexity as agents. Hence, humans show up as collectives of fundamental agents which co-evolve in an organized community. A social system is a community of communities then. (In fact, this viewpoint is also compatible with the evolutionary theory of systems according to Edgar Morin.) However, while talking about all of that, we notice that this is *itself* the outcome of the modelling procedure. In other words, the systematic approach outlined above is nothing but another model among models, i.e. a mapping of the world, not the world itself. We utilize the concepts of *space*, *network*, and *system* according to our epistemological principles: As such networks serve as a formal skeleton for a space and for a system, respectively, while they are graphical representations of both of them. The concept of space serves also the graphical representation of what we call a system. The system is the concept we have of what we are able to observe in concrete terms. But what we observe is only part of the world. (Our ontological directive is: The world is not as we observe it.) *But we are products of that world ourselves*. Hence, there is the necessity of a *cognitive metatheory* for all of our other theories which tells us something about the basic limitations of our possible knowledge. Hence also, the necessity of a self-loop: Humans model the world by inventing theories according to the cognitive constraints this same world is imposing upon humans. Theories constitute categories of meaning. If humans show up then as communities of communities of fundamental (natural) agents, they are, with respect to the latter, *emergent structures* in nature. And so are all of their reflexive concepts. Hence, the concept of (human) meaning itself is emergent with respect to fundamental *proto-meaning* defined in terms of the directed behaviour of fundamental agents in their (directed) fundamental networks of interaction. In this sense, the consciousness necessary for reflexion at all shows up as a human achievement, but on the fundamental level loops as agents may be interpreted as those who encounter the first (proto-) Big Wow – which in that case would be linked then directly to the emergence of the Universe rather than to a later inflationary period, unless one reserves Zizzi's expression for the bit period proper.

Dedication

This paper is dedicated to the memory of Alexandros Chapsiadis.

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